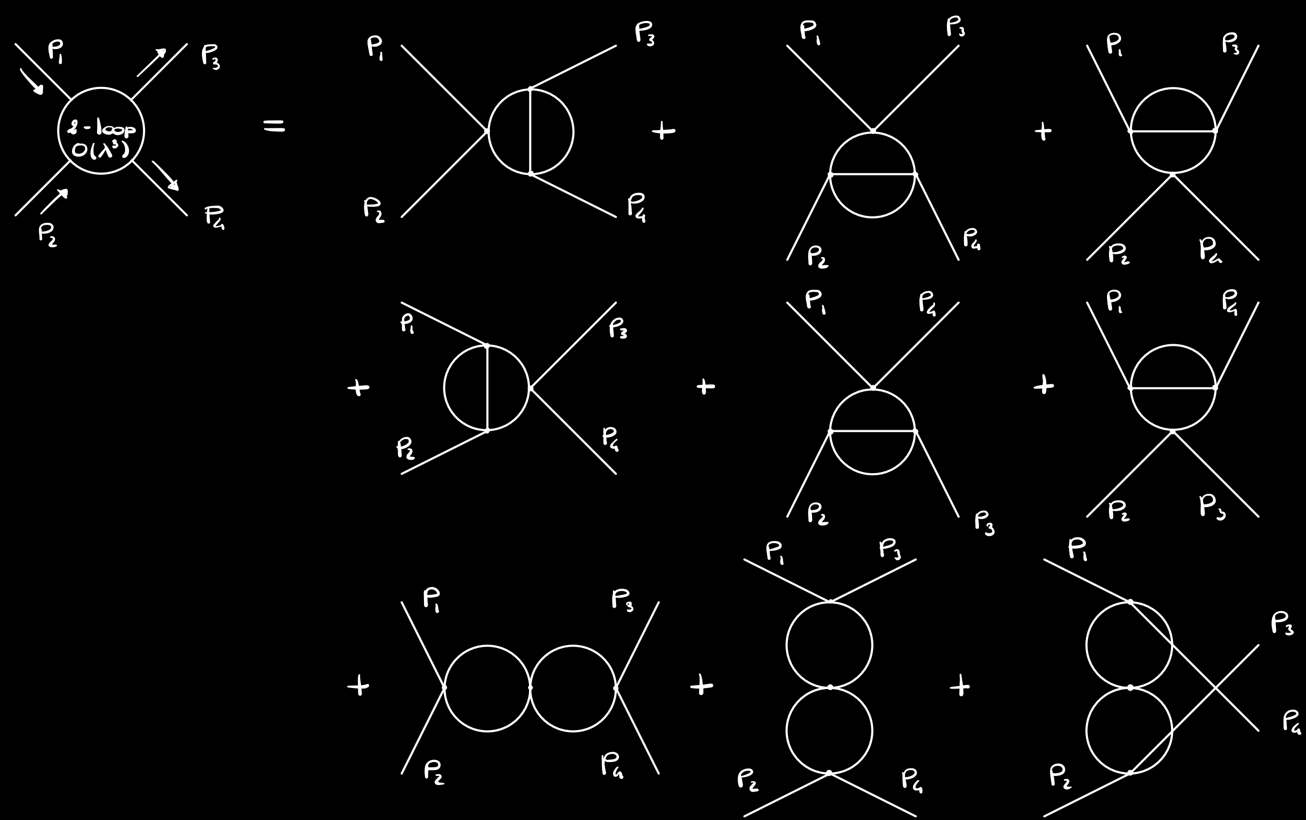
HW9

TWO LOOP RENORMALIZATION OF THE (MASSLESS) ϕ^4 THEORY

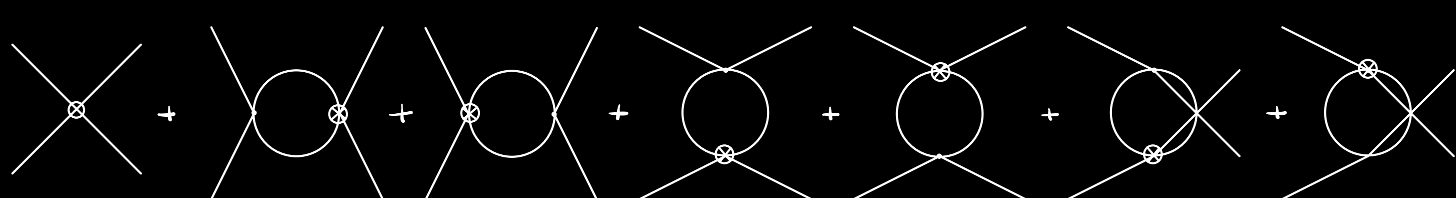
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4!}\mu^{\varepsilon}\phi^4 + \frac{\delta_Z}{2}(\partial\phi)^2 + \frac{\delta_\lambda}{4!}\mu^{\varepsilon}\phi^4$$

$$\delta_Z = O(\lambda^2)$$

$$\delta_\lambda = \frac{3}{\varepsilon}\frac{\lambda^2}{(4\pi)^2} + O(\lambda^3)$$

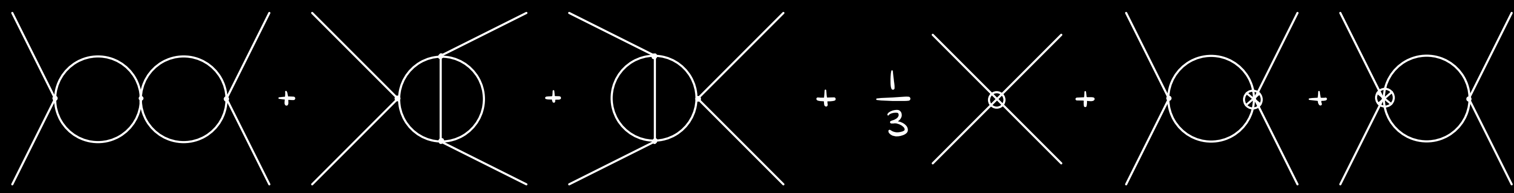


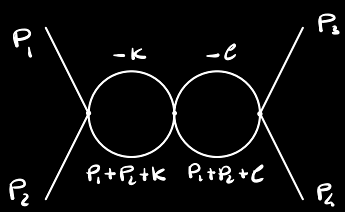
Counterterms



"s-channel"

s-channel diagrams





$$= -\frac{1}{4}\lambda^3\mu^{3\varepsilon}\int \frac{d^dk}{(2\pi)^d}\frac{d^d\ell}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{k^2}\frac{1}{(p_1+p_2+\ell)^2}\frac{1}{\ell^2} =$$

$$= -\frac{1}{4}\lambda^3\mu^{3\varepsilon}\left(\int \frac{d^dk}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{k^2}\right)^2 =$$

$$= -\frac{1}{4}\lambda^3\mu^{3\varepsilon}\left(\int_0^1 dx\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2+sx(1-x))^2}\right)^2 =$$

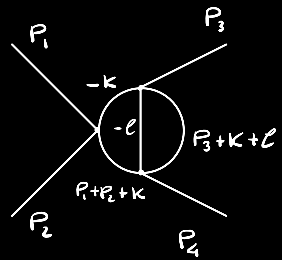
$$= -\frac{1}{4}\lambda^3\mu^{\varepsilon}\left(\frac{\mu^{\varepsilon}}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\int_0^1 dx\,\left(sx(1-x)\right)^{\frac{d}{2}-2}\right)^2 =$$

$$= -\frac{\mu^{\varepsilon}}{4}\frac{\lambda^3}{(4\pi)^4}\left[\int_0^1 dx\left(\frac{2}{\varepsilon}-\gamma-\ln\left(\frac{s}{4\pi\mu^2}\right)-\ln(x(1-x))+O(\varepsilon)\right)\right]^2 =$$

$$= -\frac{\mu^{\varepsilon}}{4}\frac{\lambda^3}{(4\pi)^4}\left[\frac{2}{\varepsilon}-\gamma+2-\ln\left(\frac{s}{4\pi\mu^2}\right)+O(\varepsilon)\right]^2 =$$

$$= -\frac{\mu^{\varepsilon}}{4}\frac{\lambda^3}{(4\pi)^4}\left[\frac{4}{\varepsilon^2}-\frac{4}{\varepsilon}\ln\left(\frac{s}{4\pi\mu^2}\right)-\frac{4}{\varepsilon}(\gamma-2)+O(\varepsilon^0)\right] =$$

$$= \mu^{\varepsilon}\frac{\lambda^3}{(4\pi)^4}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\ln\left(\frac{s}{4\pi\mu^2}\right)+\frac{1}{\varepsilon}(\gamma-2)+O(\varepsilon^0)\right]$$



$$= -\mu^{3\varepsilon}\frac{\lambda^3}{2}\int \frac{d^dk}{(2\pi)^d}\frac{d^d\ell}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{k^2}\frac{1}{(p_3+k+\ell)^2}\frac{1}{\ell^2} =$$

$$= -\mu^{3\varepsilon}\frac{\lambda^3}{2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{k^2}\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(p_3+k+\ell)^2}\frac{1}{\ell^2} =$$

$$= -\mu^{3\varepsilon}\frac{\lambda^3}{2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{k^2}\int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2+x(1-x)(p_3+k)^2)^2} =$$

$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{k^2}\left[x(1-x)(p_3+k)^2\right]^{\frac{d}{2}-2}$$

Set $p_3=0$

$$\overset{\text{div. part}}{=} -\mu^{3\varepsilon}\frac{\lambda^3}{2}\frac{1}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\int_0^1 dx\, x^{\frac{d}{2}-2}(1-x)^{\frac{d}{2}-2}\int\frac{d^dk}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{(k^2)^{3-\frac{d}{2}}} =$$

$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma(d-2)}\int\frac{d^dk}{(2\pi)^d}\frac{1}{(p_1+p_2+k)^2}\frac{1}{(k^2)^{3-\frac{d}{2}}} =$$

$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma(d-2)}\frac{\Gamma\left(4-\frac{d}{2}\right)}{\Gamma\left(3-\frac{d}{2}\right)}\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{(1-x)^{2-\frac{d}{2}}}{\left[k^2+x(1-x)s\right]^{4-\frac{d}{2}}}$$

$$\frac{1}{A^\alpha B^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_0^1 dx\,\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\left[Ax+B(1-x)\right]^{\alpha+\beta}}$$

$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^{d}}\Gamma\left(2-\frac{d}{2}\right)\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma(d-2)}\frac{\Gamma\left(4-\frac{d}{2}\right)}{\Gamma\left(3-\frac{d}{2}\right)}\frac{\Gamma(4-d)}{\Gamma\left(4-\frac{d}{2}\right)}$$

$$\times\int_0^1 dx\,(1-x)^{2-\frac{d}{2}}\left[x(1-x)s\right]^{d-4} =$$

$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^{d}}\frac{\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(d-2)}\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma\left(3-\frac{d}{2}\right)}\Gamma(4-d)\int_0^1 dx\, x^{d-4}(1-x)^{\frac{d}{2}-2}s^{d-4} =$$

$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^{d}}\frac{\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(d-2)}\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma\left(3-\frac{d}{2}\right)}\Gamma(4-d)\frac{\Gamma(d-3)\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{3}{2}d-4\right)}s^{d-4} =$$

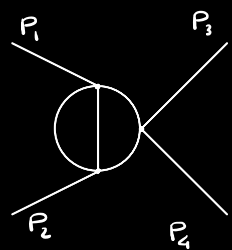
$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^d}\frac{\Gamma(2-\frac{d}{2})}{\Gamma(d-2)}\frac{\Gamma(\frac{d}{2}-1)^3}{\Gamma(3-\frac{d}{2})}\frac{\Gamma(d-3)}{\Gamma(\frac{3}{2}d-4)}\Gamma(4-d)\, s^{d-4} =$$

$$= -\frac{\lambda^3}{2}\frac{\mu^{3\varepsilon}}{(4\pi)^d}\frac{\Gamma(2-\frac{d}{2})}{(d-3)\Gamma(d-3)}\frac{\Gamma(\frac{d}{2}-1)^3}{(2-\frac{d}{2})\Gamma(2-\frac{d}{2})}\frac{\Gamma(d-3)}{\Gamma(\frac{3}{2}d-4)}(3-d)\Gamma(3-d)\, s^{d-4} =$$

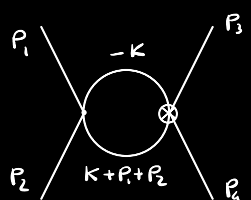
$$= \lambda^3\frac{\mu^{3\varepsilon}}{(4\pi)^d}\frac{\Gamma(\frac{d}{2}-1)^3}{4-d}\frac{\Gamma(3-d)}{\Gamma(\frac{3}{2}d-4)}\, s^{d-4} =$$

$$= \mu^\varepsilon\frac{\lambda^3}{(4\pi)^4}\left(\frac{s}{4\pi\mu^2}\right)^{-\varepsilon}\left(-\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}(5-2\gamma)+O(\varepsilon^0)\right) =$$

$$= \mu^\varepsilon\frac{\lambda^3}{(4\pi)^4}\left(-\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}(5-2\gamma)+\frac{1}{\varepsilon}\ln\left(\frac{s}{4\pi\mu^2}\right)+O(\varepsilon^0)\right)$$



$$= \mu^\varepsilon\frac{\lambda^3}{(4\pi)^4}\left(-\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}(5-2\gamma)+\frac{1}{\varepsilon}\ln\left(\frac{s}{4\pi\mu^2}\right)+O(\varepsilon^0)\right)$$



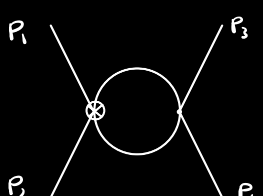
$$= \frac{1}{2}\mu^{2\varepsilon}\lambda\,\delta_\lambda\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k+P_1+P_2)^2}\frac{1}{k^2} =$$

$$= \frac{1}{2}\mu^\varepsilon\frac{\lambda\,\delta_\lambda}{(4\pi)^2}\left(\frac{2}{\varepsilon}-\gamma+2-\ln\left(\frac{s}{4\pi\mu^2}\right)+O(\varepsilon)\right)$$

$$\delta_\lambda = \frac{3}{\varepsilon}\frac{\lambda^2}{(4\pi)^2}+O(\lambda^3)$$

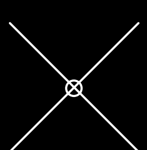
$$= \frac{3}{2}\mu^\varepsilon\frac{\lambda^3}{(4\pi)^4}\left(\frac{2}{\varepsilon^2}-\frac{1}{\varepsilon}(\gamma-2)-\frac{1}{\varepsilon}\ln\left(\frac{s}{4\pi\mu^2}\right)+O(\varepsilon^0)\right)$$

$$= \mu^\varepsilon\frac{\lambda^3}{(4\pi)^4}\left(\frac{3}{\varepsilon^2}-\frac{3}{2}\frac{1}{\varepsilon}(\gamma-2)-\frac{3}{2}\frac{1}{\varepsilon}\ln\left(\frac{s}{4\pi\mu^2}\right)+O(\varepsilon^0)\right)$$

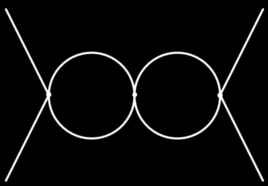


$$= \frac{1}{2}\mu^{2\varepsilon}\lambda\,\delta_\lambda\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k+P_1+P_2)^2}\frac{1}{k^2} =$$

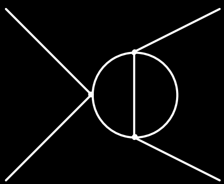
$$= \mu^\varepsilon\frac{\lambda^3}{(4\pi)^4}\left(\frac{3}{\varepsilon^2}-\frac{3}{2}\frac{1}{\varepsilon}(\gamma-2)-\frac{3}{2}\frac{1}{\varepsilon}\ln\left(\frac{s}{4\pi\mu^2}\right)+O(\varepsilon^0)\right)$$



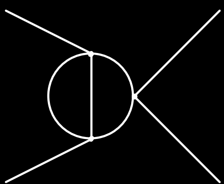
$$= -\mu^\varepsilon\delta_\lambda$$



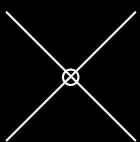
$$= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \ln\left(\frac{s}{4\pi\mu^2}\right) + \frac{1}{\varepsilon}(\gamma-2) + O(\varepsilon^0) \right]$$



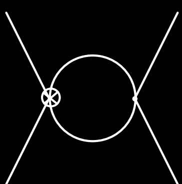
$$= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \ln\left(\frac{s}{4\pi\mu^2}\right) - \frac{1}{2\varepsilon}(5-2\gamma) + O(\varepsilon^0) \right]$$



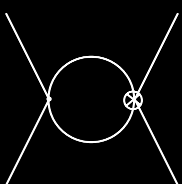
$$= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \ln\left(\frac{s}{4\pi\mu^2}\right) - \frac{1}{2\varepsilon}(5-2\gamma) + O(\varepsilon^0) \right]$$



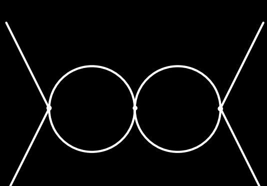
$$= -\mu^{\varepsilon} \delta_{\lambda}$$



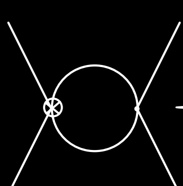
$$= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[\frac{3}{\varepsilon^2} - \frac{3}{2}\frac{1}{\varepsilon} \ln\left(\frac{s}{4\pi\mu^2}\right) - \frac{3}{2}\frac{1}{\varepsilon}(\gamma-2) + O(\varepsilon^0) \right]$$



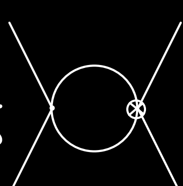
$$= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[\frac{3}{\varepsilon^2} - \frac{3}{2}\frac{1}{\varepsilon} \ln\left(\frac{s}{4\pi\mu^2}\right) - \frac{3}{2}\frac{1}{\varepsilon}(\gamma-2) + O(\varepsilon^0) \right]$$



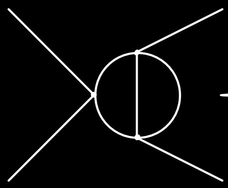
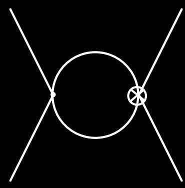
$+ \frac{1}{3}$

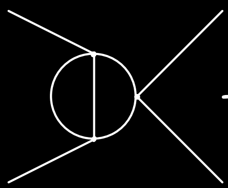
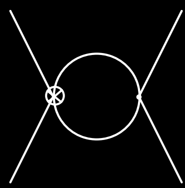


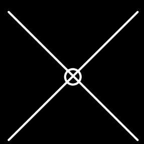
$+ \frac{1}{3}$



$$= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[\frac{1}{\varepsilon^2} + O(\varepsilon^0) \right]$$

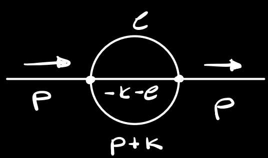
 $+ \frac{2}{3}$  $= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[\frac{1}{\varepsilon^2} - \frac{1}{2}\frac{1}{\varepsilon} + O(\varepsilon^0) \right]$

 $+ \frac{2}{3}$  $= \mu^{\varepsilon} \frac{\lambda^3}{(4\pi)^4} \left[\frac{1}{\varepsilon^2} - \frac{1}{2}\frac{1}{\varepsilon} + O(\varepsilon^0) \right]$

 $= -\mu^{\varepsilon} \delta_\lambda^{(2)}$

$$\delta_\lambda^{(2)} = \frac{9}{\varepsilon^2}\frac{\lambda^3}{(4\pi)^4} - \frac{3}{\varepsilon}\frac{\lambda^3}{(4\pi)^4}$$

$$\boxed{\delta_\lambda = \frac{3}{\varepsilon}\frac{\lambda^2}{(4\pi)^2} + \frac{9}{\varepsilon^2}\frac{\lambda^3}{(4\pi)^4} - \frac{3}{\varepsilon}\frac{\lambda^3}{(4\pi)^4} + O(\lambda^4)}$$



$$= \mu^{2\varepsilon}\frac{\lambda^2}{6}\int \frac{d^dk}{(2\pi)^d}\frac{d^d\ell}{(2\pi)^d}\frac{1}{(p+k)^2}\frac{1}{(k+\ell)^2}\frac{1}{\ell^2} =$$

$$= \mu^{2\varepsilon}\frac{\lambda^2}{6}\int\frac{d^dk}{(2\pi)^d}\int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2+x(1-x)k^2)^2}\frac{1}{(p+k)^2} =$$

$$= \frac{\mu^{2\varepsilon}}{6}\frac{\lambda^2}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\int\frac{d^dk}{(2\pi)^d}\frac{1}{(p+k)^2}\int_0^1 dx\,\left[x(1-x)k^2\right]^{\frac{d}{2}-2} =$$

$$= \frac{\mu^{2\varepsilon}}{6}\frac{\lambda^2}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma(d-2)}\int\frac{d^dk}{(2\pi)^d}\frac{1}{(p+k)^2(k^2)^{2-\frac{d}{2}}}$$

$$= \frac{\mu^{2\varepsilon}}{6}\frac{\lambda^2}{(4\pi)^{d/2}}\Gamma\left(2-\frac{d}{2}\right)\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma(d-2)}\frac{\Gamma\left(3-\frac{d}{2}\right)}{\Gamma\left(2-\frac{d}{2}\right)}\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{(1-x)^{1-\frac{d}{2}}}{\left[k^2+x(1-x)p^2\right]^{3-\frac{d}{2}}}$$

$$= \frac{\mu^{2\varepsilon}}{6}\frac{\lambda^2}{(4\pi)^{d}}\Gamma\left(2-\frac{d}{2}\right)\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma(d-2)}\frac{\Gamma\left(3-\frac{d}{2}\right)}{\Gamma\left(2-\frac{d}{2}\right)}\frac{\Gamma(3-d)}{\Gamma\left(3-\frac{d}{2}\right)}\times$$

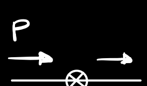
$$\times\int_0^1 dx\,(1-x)^{1-d/2}\left[x(1-x)p^2\right]^{d-3} =$$

$$= \frac{\mu^{2\varepsilon}}{6}\frac{\lambda^2}{(4\pi)^{d}}\frac{\Gamma\left(\frac{d}{2}-1\right)^2}{\Gamma(d-2)}\Gamma(3-d)\frac{\Gamma(d-2)\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{3}{2}d-3\right)}(p^2)^{d-3} =$$

$$= \frac{1}{6}\frac{\lambda^2}{(4\pi)^4}\frac{\Gamma\left(\frac{d}{2}-1\right)^3}{\Gamma\left(\frac{3}{2}d-3\right)}\Gamma(3-d)\,p^2\left(\frac{p^2}{4\pi\mu^2}\right)^{d-4} =$$

$$= \frac{1}{6}\frac{\lambda^2}{(4\pi)^4}p^2\left(-\frac{1}{2\varepsilon}+\frac{1}{8}(-13+6\gamma)+\frac{1}{2}\ln\left(\frac{p^2}{4\pi\mu^2}\right)+O(\varepsilon)\right) =$$

$$= \frac{\lambda^2}{(4\pi)^4}p^2\left(-\frac{1}{12}\frac{1}{\varepsilon}+\frac{1}{48}(-13+6\gamma)+\frac{1}{12}\ln\left(\frac{p^2}{4\pi\mu^2}\right)+O(\varepsilon)\right)$$

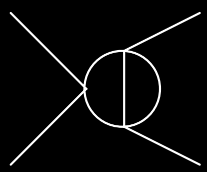


$$= -\delta_Z\, p^2$$

if we identify

$$\delta_Z = -\frac{1}{12\varepsilon}\frac{\lambda^2}{(4\pi)^4} + O(\lambda^3)$$

Alternative way



$$= -\mu^{3\varepsilon}\,\frac{\lambda^3}{2}\int \frac{d^dk}{(2\pi)^d}\,\frac{d^d\ell}{(2\pi)^d}\,\frac{1}{(P_1+P_2+k)^2}\,\frac{1}{k^2}\,\frac{1}{(P_3+k+\ell)^2}\,\frac{1}{\ell^2}$$

$$= -\frac{\lambda^3}{2}\,\frac{\mu^{3\varepsilon}}{(4\pi)^{d/2}}\,\Gamma(2-d/2)\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\,\frac{1}{(P_1+P_2+k)^2}\,\frac{1}{k^2}\,\left[x(1-x)(P_3+k)^2\right]^{\frac{d}{2}-2}$$

$$= -\frac{\lambda^3}{2}\,\frac{\mu^{3\varepsilon}}{(4\pi)^{d/2}}\,\Gamma(2-d/2)\,\frac{\Gamma(\frac{d}{2}-1)^2}{\Gamma(d-2)}\int\frac{d^dk}{(2\pi)^d}\,\frac{1}{(P_1+P_2+k)^2}\,\frac{1}{k^2((P_3+k)^2)^{2-d/2}}$$

$$I = \int\frac{d^dk}{(2\pi)^d}\,\frac{1}{(P_1+P_2+k)^2}\,\frac{1}{k^2}\,\frac{1}{(P_3+k)^{2-d/2}}$$

$$= \frac{\Gamma(4-d/2)}{\Gamma(2-d/2)}\int dx\,dy\,dz\;\delta(1-x-y-z)\;z^{1-d/2}$$

$$\int\frac{d^dk}{(2\pi)^d}\,\frac{1}{\left[x(k+(P_1+P_2))^2+yk^2+z(P_3+k)^2\right]^{4-d/2}}$$

$$(x+y+z)k^2 + 2k\cdot(P_1+P_2)x + xs + zP_3^2 + z\,2P_3\cdot k$$

$$(k+x(P_1+P_2)+zP_3)^2 - x^2s - z^2P_3^2 - 2xz\,P_3\cdot(P_1+P_2)$$

$$+xs + zP_3^2$$

$$= (k+x(P_1+P_2)+zP_3)^2 + \Delta$$

$$\Delta = x(1-x)P_{12}^2 + z(1-z)P_3^2 - 2xz\,P_3\cdot P_{12}$$

$$I = \frac{\Gamma(4-d/2)}{\Gamma(2-d/2)} \int dx\,dy\,dz\; \delta(1-x-y-z)\; z^{1-d/2}$$

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+\Delta)^{4-d/2}}$$

$$\frac{\Omega_d}{2} \frac{(\Delta)^{\frac{d}{2}+\frac{d}{2}-4}}{(2\pi)^{d/2}} \frac{\Gamma(d/2)\,\Gamma(4-d)}{\Gamma(4-d/2)}$$

$$= \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(4-d)}{\Gamma(2-d/2)}$$

$$\int dx\,dy\,dz\; \delta(1-x-y-z)\; z^{1-d/2}$$

$$(x(1-x)P_{12}^2 + z(1-z)P_3^2 - 2xz\; P_3\cdot P_{12})^{d-4}$$

$$x P_{12}^2 + z P_3^2 - (xP_{12} + zP_3)^2$$

The singularity is in $z=0$.

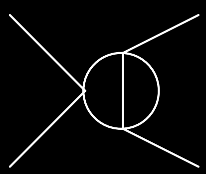
$$z^{1-d/2} f(z) = z^{1-d/2} f(0) + \underset{\substack{\uparrow \\ \text{Finite}}}{z^{2-d/2}} f'(0) + \ldots$$

$$I \overset{\text{div}}{=} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(4-d)}{\Gamma(2-d/2)} \int_0^1 dx \int_0^{1-x} dz\; z^{1-d/2}\; x^{d-4}(1-x)^{d-4}(P_{12}^2)^{d-4}$$

$$= \frac{(S)^{d-4}}{(4\pi)^{d/2}} \frac{\Gamma(4-d)}{\Gamma(2-d/2)} \int_0^1 dx\; \frac{2}{4-d}\; (1-x)^{2-d/2}\; x^{d-4} (1-x)^{d-4}$$

$$= \frac{S^{d-4}}{(4\pi)^{d/2}} \frac{\Gamma(4-d)}{\Gamma(2-d/2)} \frac{2}{4-d} \int_0^1 dx\; x^{d-4} (1-x)^{\frac{d}{2}-2}$$

$$= \frac{s^{d-4}}{(4\pi)^{d/2}} \frac{\Gamma(4-d)}{\Gamma(2-d/2)} \frac{2}{4-d} \frac{\Gamma(d-3)\,\Gamma(\frac{d}{2}-1)}{\Gamma(\frac{3}{2}d-4)}$$



$$= -\frac{\lambda^3}{2} \frac{\mu^{3\varepsilon}}{(4\pi)^{d/2}} \Gamma(2-d/2) \frac{\Gamma(\frac{d}{2}-1)^2}{\Gamma(d-2)}$$

$$\frac{s^{d-4}}{(4\pi)^{d/2}} \frac{\Gamma(4-d)}{\Gamma(2-d/2)} \frac{2}{4-d} \frac{\Gamma(d-3)\,\Gamma(\frac{d}{2}-1)}{\Gamma(\frac{3}{2}d-4)}$$

$$= -\frac{\lambda^3}{2} \frac{\mu^{3\varepsilon}}{(4\pi)^{d}} \frac{2}{4-d} \frac{\Gamma(\frac{d}{2}-1)^3}{\Gamma(\frac{3}{2}d-4)} \frac{\overset{(3-d)\,\Gamma(3-d)}{\overset{\shortparallel}{\Gamma(4-d)}}\,\Gamma(d-3)}{\Gamma(d-2) = (d-3)\,\Gamma(d-3)} s^{d-4}$$

$$= \lambda^3 \frac{\mu^{3\varepsilon}}{(4\pi)^{d}} \frac{\Gamma(3-d)}{4-d} \frac{\Gamma(\frac{d}{2}-1)^3}{\Gamma(\frac{3}{2}d-4)} s^{d-4}$$